Problem 6 (10 points). Let ℓ_0 be the subspace of ℓ^2 consisting of all sequences $x = (x_n)_{n=1}^{\infty}$ which eventually are zero, cf. Example 2.7 in the textbook. Equip ℓ^2 and ℓ_0 with the norm $\|x\| = (x, x)^{1/2}$ arising from the usual inner product on ℓ^2.

Consider the linear functionals $F, G \colon \ell_0 \to \mathbb{C}$ given by

$$F(x) = \sum_{n=1}^{\infty} x_n, \qquad G(x) = \sum_{n=1}^{2011} x_n, \quad x = (x_n)_{n=1}^{\infty} \in \ell_0.$$

Show that G is bounded and that F is unbounded. Determine $\|G\|$.